RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.SC. FIRST SEMESTER (July – December) 2014

Mid-Semester Examination, September 2014

Date : 15/09/2014 **PHYSICS (Honours)**

Time : 11 am – 1 pm Paper : I Full Marks : 50

[Use a separate answer book for each group]

[*Answer **five questions** taking at least **one** from each group*]

Group – A

1. a) What do you mean by singular matrix. [1]
   b) Show that A is singular matrix if and only if it has zero eigenvalue. [5]
   c) Prove that $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$. [4]

2. a) Consider the following transformation in three dimensions.
   $x' = x\cos\theta + y\sin\theta,\ y' = -x\sin\theta + y\cos\theta,\ z' = z$
      i) Write down the transformation matrix $A(\theta)$. [1]
      ii) Show that $A(\theta_1)A(\theta_2) = A(\theta_1 + \theta_2)$. [3]
      iii) Show that $A(\theta)$ is orthogonal matrix. [2]
   b) Find the eigen value and eigenvector of the matrix $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$. [4]

Group – B

3. a) Write down the condition of linear independence of a set of vectors in n-dimensions. Hence test if the three vectors $\bar{A} = 3\bar{i} - 2\bar{j} + 4\bar{k}$, $\bar{B} = \bar{i} + 3\bar{j} + \bar{k}$ and $\bar{c} = 5\bar{i} + 6\bar{k}$ are linearly independent in three dimensions. [5]
   b) Construct a set of three orthonormal vectors from the following three vectors $\bar{A} = 2\bar{i} - 3\bar{j} + 4\bar{k}$, $\bar{B} = \bar{i} + \bar{j} + \bar{k}$ and $\bar{C} = 5\bar{i} - 6\bar{j} + 2\bar{k}$ by Gram-Schmidt orthgonalisation method. [5]

4. a) If $\phi = 5x^2 + 7xy + 8y^2z$, calculate the directional derivative at the point (1,1,1) along the direction $3\bar{i} + 2\bar{j} + \bar{k}$. Along which direction is this maximum? [4]
   b) Test if the following vectors are conservative. [6]
      i) $\vec{F} = \bar{i}yz + \bar{j}zx + \bar{k}xy$
      ii) $\vec{F} = \bar{i}xy + \bar{j}yz + \bar{k}zx$

Group – C

5. a) Define an inertial frame of reference. A space-time event is observed from two inertial frames S and S′. If S′ is in uniform translational motion relative to S, obtain the transformation equations between the space-time coordinates of the event in S and S′. (Galilean transformations) [3]
   b) A particle moves n a trajectory such that at time t, its polar coordinates are
   $r = \frac{bt}{\tau^2}(2\tau - t), \theta = \frac{t}{\tau}(0 \le t \le 2\tau)$
   where b and τ are positive constants. Find the velocity vector of the particle at time t. [3]

c) Derive the work-energy theorem for a particle of constant mass moving under a force field $\vec{F}$. Show that if $\vec{F}$ is conservative, a potential energy function V can be defined such that the total mechanical energy is conserved. [4]

6. a) A particle of mass m is projected vertically upward with an initial velocity u_0. A resistive force of magnitude $R = mku^2 \ (k > 0, \text{const})$ acts on the particle

   i) Set up the equation of motion during ascent, in vector form.

   ii) Use (i) to find the height of rise when the particle has velocity v

   iii) Hence, find the maximum height reached. [1+3+1]

   b) i) Set up the differential equation of motion of a body whose mass is changing during motion, and interpret it physically.

   ii) Use (i) to obtain the equation of motion of a rocket. [3+2]

Group – D

7. a) State Fermat's principle. Show that if a point source is placed at one of the focii of an ellipsoidal mirror, the reflected rays must meet at other focus. [1+2]

   b) A light ray is refracted at a curved surface of radius of curvature R. The refractive index of the medium on the incident side is n_1 and that on the other side is n_2. Show by Fermat's principle that $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ where u and v are object and image distances respectively. [4]

   c) Light takes 0·02 see to move over the distance between two points A, B in vaccum. The space is next filled with a liquid of r, i, 1·33. Calculate the time taken by light to travel between the same two points now. What is the principle of reversibility of light? [2+1]

8. a) Show that the distance between first principal point and nodal point is equal to distance between second principal point and nodal point. [3]

   b) Two convex lenses of focal lengths 2cm and 6 cm are placed 4cm apart. Find the cardinal points of the system by matrix method. Show the points in a diagram. [4]

   c) Show that the plane surface of a plano convex or plano concave lens does not contribute to the system matrix. What are aplanatic points? [2+1]

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